An application of receding-horizon neural control in humanoid robotics

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Abstract : Optimal trajectory planning of a humanoid arm is addressed. The reference setup is the humanoid robot James [4]. The goal is to make the end effector reach a desired target or track it when it moves in the arm's workspace unpredictably. Physical constraints and setup capabilities prevent us to compute the optimal control online, so an off-line explicit control is required. Following previous studies [1], a receding-horizon method is proposed that consists in assigning the control function a fixed structure (e.g., a feedforward neural network) where a fixed number of parameters have to be tuned. More specifically a set of neural networks (corresponding to the control functions over a finite horizon) is optimized using the Extended Ritz Method. The expected value of a suitable cost is minimized with respect to the free parameters in the neural networks. Therefore, a nonlinear programming problem is addressed that can be solved by means of a stochastic gradient technique. The resulting approximate control functions are sub-optimal solutions, but (thanks to the well-established approximation properties of the neural networks) one can achieve any desired degree of accuracy [5]. Once solved the off-line finite-horizon problem, only the first control function is retained in the on-line phase: at any sample time t, given the system's state and the target's position and velocity, the control action is generated with a very small computational effort.

1 Introduction

In robotics, the task of positioning the end effectors is fundamental: whenever a robot has to move its arm in order to grasp an object, track a moving target, avoid collision with the environment or just explore it, reaching is involved. Given the target position, estimated for example by a vision system, it is common practice to plan a suitable trajectory in the cartesian space and then to find the corresponding joint and torque commands. In industrial robotics, trajectories usually have a parametrized but fixed structure, e.g. splines or polynomials, or motor commands can be found analytically after the minimization of some Lyapunov function describing the reaching goal. In humanoid robotics, the focus is not only on reaching the target, but on how the target is reached, that is the criterion which a certain limb accomplishes while performing a movement

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or acting on the environment. One of the main goals of humanoid robotics is indeed to reproduce the behaviors emerging in human motion. It is common belief that the human body moves "optimally" with respect to different cost functions, depending on action, limbs, task. In order to give a humanoid robot the chance to implement different motion criteria, it is necessary to provide a technique which consent to find optimal controls for any given cost function. To this end, a Finite Horizon (FH) optimal control problem can be addressed, but it is scarcely useful as generally the duration of the movements cannot be predicted *a priori*. Moreover, moving through a fixed horizon strategy could lead to a lack of responsiveness, whenever the target dynamics is too fast and no previous information about it are available to make some prediction. A Receding Horizon (RH) approach is then suggested. Within the classical RH approach, at each time instant t, when the system state is $\mathbf{x}_t$ a FH optimal control problem is solved and a sequence of N optimal control actions is computed, $\mathbf{u}^{FH}_{0|t}, \mathbf{u}^{FH}_{1|t}, \ldots, \mathbf{u}^{FH}_{N-1|t}$ (corresponding to velocity, acceleration or torque commands, depending on the control design), which minimize a suitable cost function affecting the motion performances; then only the first control vector is applied: $\mathbf{u}^{RH}_t = \mathbf{u}^{FH}_{0|t}$. This procedure is repeated stage after stage, thus yielding a feedback control law. Stabilizing properties of RH control have been shown for case of both linear and nonlinear, continuous and discrete time systems, using the terminal equality constraints $\mathbf{x}_{t+N} = \mathbf{0}$ [8], relaxing it [9] and just imposing the attractiveness of the origin by means of penalty functions [1].

The classical RH technique assumes the control vectors to be generated after the solution of a nonlinear programming problem at each time instant: this assumption is generally unrealistic in the case of humanoid robotics, as the robot's and the target's dynamics are very fast. In order to solve the optimization problem on-line, with the guarantee not to violate any temporal constraint, a proper hardware and software are required, usually a real-time processing unit supporting fast and highly precise computations, directly connected to the robot' sensing and actuation devices. Unfortunately, different multi-level control architectures often do not support this control scheme. This is the case of our humanoid robot, James [4]. James is a humanoid torso, with the overall size of a 10 years old boy, a total weight of about 8 kg, and a 7 DOF arm (3-shoulder, 1-elbow, 3-wrist). Torque is transmitted to the joints by plastic toothed belts, pulleys and stainless-steel tendons, actuated by rotary motors. The robot's motion can be controlled by sending position and velocity commands from a remote PC to 12 Digital Signal Processing (DSP) boards (Freescale DSP56F807, 8MHz, fixed point 16 bit number representation), via CAN bus. DSP boards have limited memory and computation capability and cannot support more than simple operations, namely low level motor control (mostly PID controllers), signal acquisition and pre-filtering from the encoders. For this reason, implementing an on-line controller is impossible in the current setup: an explicit off-line RH controller is then proposed.

The goal of this work is to design a feedback RH regulator for reaching tasks, that must reveal itself to be quick and reactive to changes, in particular able to track a target moving in the robot's workspace in an unpredictable way. We will also describe a technique which concentrates in an off line phase the computation of a time-invariant feedback optimal control law, for every possible system and target states belonging to the set of admissible ones. The proposed

algorithm consists of two steps. In the first one, a suitable sequence of neural networks is trained off line, so that they can approximate the optimal solutions of a stochastic FH control problem, which is generalized for every possible state configuration. In the second (online phase), only the first control law is applied, at each time instant. The Extended RItz Method (ERIM) [6] is chosen as a functional approximation technique. The use of feedforward neural networks (thanks to their well known approximation capabilities [7]) guarantees that the optimal solutions can be approximated at any desired degree of accuracy. We remark that the computation is concentrated in the off line phase, while in the on line phase only the computation of the current control is performed, thus yielding a quick response to unpredictable changes in the target's state. The feasibility of this approach has already been tested on the control of a nonholonomic robot [2].

James robot's model is an open kinematic chain. In the following we shall only focus on the arm motion control, in particular from the shoulder up to the wrist, which will be considered as the end effector of the kinematic chain, and neglect the rotation of the hand. Let us denote by $\mathbf{x}_c^r$ the cartesian coordinates of the end effector in the robot's workspace, with respect to a fixed reference frame, and by $\mathbf{q}$ the vector whose components are the joints' coordinates of the arm. Then the forward kinematics $\mathbf{x}_c^r = f_{\text{arm}}(\mathbf{q})$, $f_{\text{arm}} : \mathbb{R}^{n_q} \rightarrow \mathbb{R}^{n_c}$, can be easily found by Denavit-Hartenberg convention [3]. In the following, we will assume the robot model to be known and kinematic singularities be avoided such that the jacobian matrix $J(\mathbf{q}) = \partial f_{\text{arm}}(\mathbf{q})/\partial \mathbf{q}$, being $\dot{\mathbf{x}}_c^r = J(\mathbf{q})\dot{\mathbf{q}}$, is always non-singular. We shall denote by $\mathbf{x}_t^r$ and $\mathbf{x}_t^g$ the robot's end effector state vector and the target's one at time instant t. We denote by $\mathbf{x}_t$, at time instant t, the difference between the end effector and the target cartesian coordinates and velocities ($\mathbf{x}_t \triangleq \mathbf{x}_t^g - \mathbf{x}_t^r$). We remark that once the optimal control $\mathbf{u}_t^\circ$ is found, then the optimal velocity controls in the joint space can be easily computed with standard formulations, i.e., $\dot{\mathbf{q}}_t^\circ = J^\#(\mathbf{q}_t)\dot{\mathbf{x}}_t^{r\circ}$, where $J^\#$ denotes the Moore-Penrose pseudo-inverse of the jacobian matrix. In particular, as explained in the previous section, they are computed by a common PC, then sent through the CAN bus to the DSP cards, where a low level control loop is performed. The control scheme is shown in Figure 1.

2 Receding horizon regulator: a neural approach

The goal of the reaching control problem is to find, at any time instant t, the optimal control $\mathbf{u}_t^\circ$ minimizing a suitable cost function, which is chosen so as to characterize the trajectories of the end effector reaching or tracking a target moving unpredictably in the robot workspace. Now let us represent the previous equations in the more general and compact form

$$\mathbf{x}_{t+1} = \mathbf{f}\left(\mathbf{x}_t, \mathbf{u}_t\right) \ , \ t = 0, 1, \ldots \tag{1}$$

where at the time instant t, $\mathbf{x}_t$ is the state vector, taking values from a finite set $X \subseteq \mathbb{R}^n$, and $\mathbf{u}_t$ is the control vector, constrained to take values from a finite set $U \subseteq \mathbb{R}^m$. At any time instant t, the desired state is $\mathbf{x}_t^* = 0$, meaning that the goal is to bring the difference between the end effector and the target to zero. By making this assumption, we implicitly apply a certainty equivalence principle: at time instant t, the target vector $\mathbf{x}^g$ is supposed to remain constant

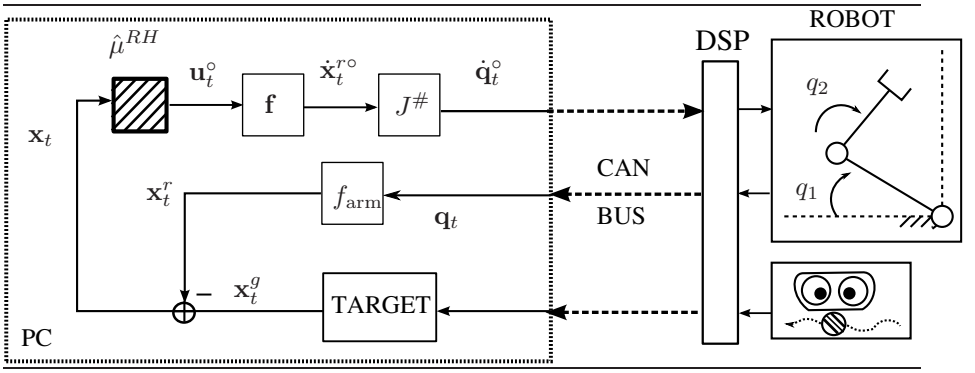

Figure 1: James's arm control scheme. Velocity commands are sent through a CAN bus, while direct motor control is performed by DSP cards. The retrieving of the target's cartesian coordinates is not modeled, as it would require to discuss the robotic visual system. The arm kinematic model is reported. We indicated two arm joints (q_1, q_2), corresponding to the case of a 2 DOF arm ($n_q = 2$).

for N time instants, that is: $\mathbf{x}^g_{t+i+1} = \mathbf{x}^g_{t+i}, i = 0, \ldots, N-1$. We can now state a RH control problem.

Problem 1. *At every time instant $t \geq 0$, find the RH optimal controls $\mathbf{u}_t^\circ \in U$, where $\mathbf{u}_t^\circ$ is the first vector of the control sequence $\mathbf{u}^\circ_{0|t}, \ldots, \mathbf{u}^\circ_{N-1|t}$ that minimize the FH cost functional*

$$\mathcal{J}(\mathbf{x}_t) = \left\{ \sum_{i=0}^{N-1} h_i(\mathbf{x}_{t+i}, \mathbf{u}_{i|t}) + h_N(\mathbf{x}_{t+N}) \right\} . \tag{2}$$

The classical RH control assumes that at each time instant of control a FH control problem is solved, and a sequence of N optimal controls is found. As we previously discussed, this approach is not suitable in our case, for the hardware limitations imposed by the DSP cards. Therefore we will change the problem's formulation so as to be able to compute the control laws in an off line phase.

Problem 2 (RH)**.** *For every time instant $t \geq 0$, find the RH optimal control law $\mathbf{u}_t^\circ = \mu_t^\circ(\mathbf{x}_t) \in U$, where μ_t° is the first control function of the sequence $\mu^\circ_{0|t}, \ldots, \mu^\circ_{N-1|t}$ that minimize the FH cost functional* [1]

$$\bar{\mathcal{J}}_t = \underset{\mathbf{x}_t \in X}{E} \left\{ \sum_{i=0}^{N-1} h_i(\mathbf{x}_{t+i}, \mu_{i|t}(\mathbf{x}_{t+i})) + h_N(\mathbf{x}_{t+N}) \right\} \tag{3}$$

Thanks to the time invariance of the systems dynamics and of the cost function, $t = 0$ can be considered as a generic time instant. Then, a single (functional) FH optimization problem is addressed.

[1] Hereinafter, the notation $\underset{\xi}{E}\{g(\xi)\}$ means the expectation of function g with respect to the stochastic variable ξ.It is important to notice that in Problem 1 the expectation is not necessary, because it is a deterministic problem.

Problem 3 (FH)**.** *Find a sequence of optimal control functions* $\mu_0^\circ, \ldots, \mu_{N-1}^\circ$*, that minimize the cost functional*

$$\bar{\mathcal{J}} = \underset{\mathbf{x}_0 \in X}{E} \left\{ \sum_{i=0}^{N-1} h_i(\mathbf{x}_i, \mu_i(\mathbf{x}_i)) + h_N(\mathbf{x}_N) \right\} \tag{4}$$

subject to the constraints $\mu_i^\circ \in \mathbf{U} \subseteq \mathbb{R}^m$ *and* $\mathbf{x}_{i+1} = \mathbf{f}(\mathbf{x}_i, \mu_i(\mathbf{x}_i))$*.*

The RH control strategy will correspond to use μ_0° as a time invariant control function, i.e., to apply $\mathbf{u}_t^{RH} = \mu^{RH}(\mathbf{x}_t) = \mu_0^\circ(\mathbf{x}_t)$.

2.1 From a functional optimization problem to a nonlinear programming one

In order to solve Problem FH we shall apply the ERIM [6], by which the functional optimization problem cane be transformed into a nonlinear programming one. More specifically, we constrain the admissible control functions $\mu_0, \mu_1, \ldots, \mu_{N-1}$ to take on a fixed parametrized structure, in the form of one-hidden-layer (OHL) networks:

$$\hat{\mu}_i(\mathbf{x}_i, \omega_i) = col \left[\sum_{h=1}^{\nu} c_{hj} \varphi_h(\mathbf{x}_i, \kappa_h) + b_j \right] \tag{5}$$

where $\hat{\mu}_i(\cdot, \omega_i) : \mathbb{R}^n \times \mathbb{R}^{(n+1)\nu+(\nu+1)m} \mapsto \mathbb{R}^m$, $c_{hj}, b_j \in \mathbb{R}, \kappa_h \in \mathbb{R}^k, j = 1, \ldots, m$, being ν the number of *neurons* constituting the network. By substituting (5) into (4), calling ω_i the parameters of the i-th OHL network $\hat{\mu}_i(\mathbf{x}_i, \omega_i)$, the general functional cost $\bar{\mathcal{J}}(\mu_0, \mu_1, \ldots, \mu_{N-1})$ is turned into a function $\hat{\mathcal{J}}_\nu(\omega)$ which is only dependent on a finite number of real parameters, $\omega = col(\omega_i, i = 0, 1, \ldots, N-1)$. We can now restate Problem 3 as:

Problem 4 (FH_ν)**.** *Find the optimal vectors of parameters* $\omega_0^\circ, \ldots, \omega_{N-1}^\circ$ *that minimize the cost function*

$$\hat{\mathcal{J}}_\nu = \underset{\mathbf{x}_0 \in X}{E} \left\{ \sum_{i=0}^{N-1} h_i(\mathbf{x}_i, \hat{\mu}_i(\mathbf{x}_i, \omega_i)) + h_N(\mathbf{x}_N) \right\} \tag{6}$$

subject to the constraints $\hat{\mu}_i(\mathbf{x}_i, \omega_i) \in \mathbf{U} \subseteq \mathbb{R}^m$ *and* $\mathbf{x}_{i+1} = \mathbf{f}(\mathbf{x}_i, \hat{\mu}_i(\mathbf{x}_i, \omega_i))$*.*

Then, for every time instant t, the time-invariant RH control law corresponds to $\mathbf{u}_t^{RH} = \hat{\mu}^{RH}(\mathbf{x}_t, \omega_0^\circ) = \hat{\mu}_0^\circ(\mathbf{x}_t, \omega_0^\circ)$.

2.2 Solution of the nonlinear programming problem by stochastic gradient

The optimal parameters in the OHL control functions can be found by a usual gradient algorithm, i.e.

$$\omega_i(k+1) = \omega_i(k) - \alpha(k) \nabla_{\omega_i} \underset{\{\mathbf{x}_0\}}{E} \left\{ \hat{\mathcal{J}}_\nu \left[\omega(k), \mathbf{x}_0 \right] \right\}, \; k = 0, 1, \ldots \; . \tag{7}$$

Within this context, it is impossible to calculate exactly all the gradient components, because of the stochastic nature of $\mathbf{x}_0$; then, instead of the gradient

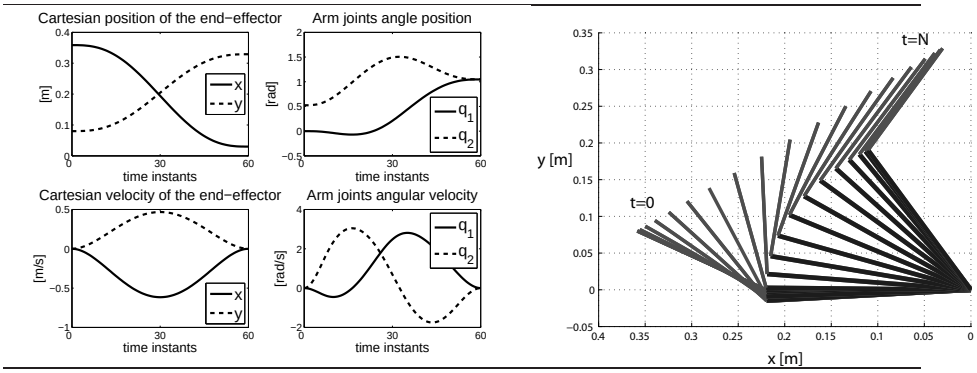

Figure 2: A minimum jerk movement of James'arm: cartesian and joints position and velocity are shown, as well as samples of the planar trajectory. The neural approximation and the analytical solution [11] coincide (m.s.e. $\cong 10^{-7}$).

$\nabla_\omega E\left[\hat{\mathcal{J}}_\nu(\omega, \mathbf{x}_0)\right]$ a single "realization" $\nabla_\omega \hat{\mathcal{J}}_\nu(\omega, \mathbf{x}_0(k))$ is computed, where the stochastic variable $\mathbf{x}_0$ is generated randomly according to its known probability density function. Then a simple gradient steepest descent algorithm can be applied:

$$\omega_i(k+1) = \omega_i(k) - \alpha(k)\nabla_{\omega_i}\hat{\mathcal{J}}_\nu\left[\omega(k), \mathbf{x}_0(k)\right] + \eta(\omega_i(k) - \omega_i(k-1)) \quad (8)$$

for $k = 0, 1, \ldots$, where we added a regularization term, weighted by $\eta \in [0, 1]$, as it is usually done when training neural networks. The convergence of the method, which is known as *stochastic gradient*, is assured by a particular choice of the step size $\alpha(k)$, that must fulfill a set of conditions [10]. Of course, one has to compute the partial derivatives of the cost $\hat{\mathcal{J}}_\nu$ with respect to the parameters to be optimized, ω_i:

$$\frac{\partial \hat{\mathcal{J}}_\nu}{\partial \omega_i} = \frac{\partial \hat{\mathcal{J}}_\nu}{\partial \mathbf{u}_i}\frac{\partial \hat{\mu}_i(\mathbf{x}_i, \omega_i)}{\partial \omega_i} \ . \quad (9)$$

The proposed algorithm for the computation of the optimal parameters consists in two phases, a forward and a backward one, and in a backpropagation technique. In the *forward phase* we "unroll" the system and the neural controllers in time, making the feedback explicit. At iteration step k, given the initial state $\mathbf{x}_0$, we compute all the state and controls generated by the sequence of OHL networks that is $\mathbf{u}_i = \hat{\mu}_i(\mathbf{x}_i, \omega_i(k))$, given $\mathbf{x}_0, \mathbf{x}_i = f(\mathbf{x}_{i-1}, \mathbf{u}_{i-1})$, $i = 1, \ldots, N$. Then we can compute all the partial costs $h_i(\mathbf{x}_i)$, $h_N(\mathbf{x}_N)$. In the *backward phase*, we compute all the gradient components and "back-propagate" them through the networks' chain. The recursive propagation is described by the following equations, for $i = N-1, N-2, \ldots, 0$:

$$\frac{\partial \hat{\mathcal{J}}_\nu}{\partial \mathbf{u}_i} = \frac{\partial h_i(\mathbf{x}_i, \mathbf{u}_i)}{\partial \mathbf{u}_i} + \frac{\partial \hat{\mathcal{J}}_\nu}{\partial \mathbf{x}_{i+1}}\frac{\partial f(\mathbf{x}_i, \mathbf{u}_i)}{\partial \mathbf{u}_i} \quad (10)$$

$$\frac{\partial \hat{\mathcal{J}}_\nu}{\partial \mathbf{x}_i} = \frac{\partial h_i(\mathbf{x}_i, \mathbf{u}_i)}{\partial \mathbf{x}_i} + \frac{\partial \hat{\mathcal{J}}_\nu}{\partial \mathbf{x}_{i+1}}\frac{\partial f(\mathbf{x}_i, \mathbf{u}_i)}{\partial \mathbf{x}_i} + \frac{\partial \hat{\mathcal{J}}_\nu}{\partial \mathbf{u}_i}\frac{\partial \hat{\mu}_i(\mathbf{x}_i, \omega_i)}{\partial \mathbf{x}_i} \quad (11)$$

initialized by $\partial \hat{\mathcal{J}}_\nu / \partial \mathbf{x}_N = \partial h_N(\mathbf{x}_N)/\partial \mathbf{x}_N$.

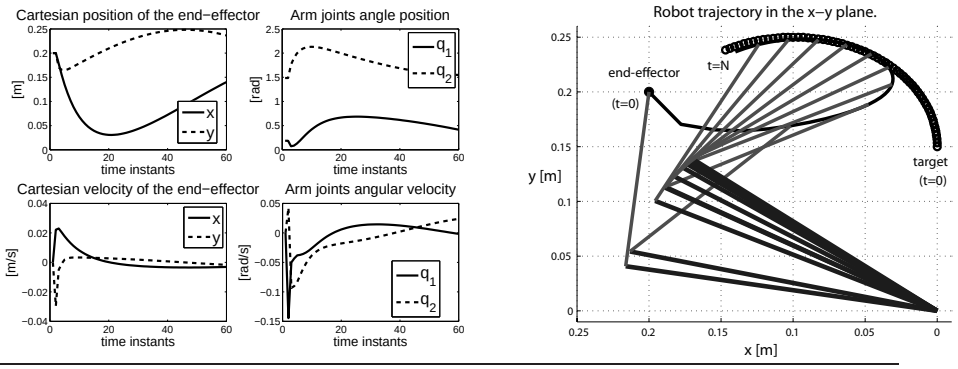

Figure 3: James' left end effector tracking a target moving in an unpredictable way, according to cost function (13), where $V_i = diag(1.0, 80.0, 5.0, 10.0)$, $V_N = 40I$. Moreover, $N = 30, \nu = 40$.

3 Results

Many neurological studies investigate the arm motion on planar surfaces, considering a two-rotative joints limb. In this case, it has been proved that the human arm moves according to *minimum jerk* principle [11]:

$$\mathcal{J} = \int_0^T \left[\left(\frac{d^3 x^r}{dt^3} \right)^2 + \left(\frac{d^3 y^r}{dt^3} \right)^2 \right] dt \ . \tag{12}$$

This criterion has been chosen to verify the effectiveness of the proposed method. We set $n_q = n_c = 2$ to consider James' arm as a two-link rigid body, moving on a planar surface, $T = 60$, $\nu = 40$, and used approximatively 10^9 samples for the off-line training of the neural networks. Results are shown in Figure 2. The method has been also tested with a different cost function:

$$\mathcal{J} = \sum_{i=t}^{t+N-1} \mathbf{c}(\mathbf{u}_i) + \mathbf{x}_{i+1}^T V_{i+1} \mathbf{x}_{i+1} \tag{13}$$

where the criterion for the task accomplishment is a tradeoff between the minimization of the energy consumption (for physical limits, it is important not to exceed in current absorption) and the "best" end-effector proximity to the target during and at the end of the manoeuvre (it could not be able to reach it perfectly, as a consequence of the unpredictable behavior of the target or the robot's intrinsic physical limits). Weight matrices V_i are chosen such as to obtain reasonable compromise between the attractiveness of the target and the energy consumption, whereas $c(u_t^j), j = x, y$ is a nonlinear but convex function (the same reported in [2]). An example of a RH trajectory during a tracking task are shown in Figure 3.

We remark that the constraints on the admissible values of $\mathbf{x}_t$ and $\mathbf{u}_t$ are always verified. To be more precise, the classical OHL networks were slightly modified, specifically by adding two sigmoidal functions $\sigma(z) = \tanh(z)$, bounded by the values $[-U, U]$, to the final output layer: with this choice, the constraints on the control values can be removed from the problem formulation since the neural networks already embed them.

4 Conclusion

The computation of the neural time invariant feedback control law is concentrated in the off line phase. Hence, at any time instant t only the first computed neural network is used. This approach is efficient in real time, because the computation of the new control action is quick, consisting only in few mathematical operations. We point out that the property of computing controls in real time as fast as possible is a strict requirement, because the integration with a visual feedback is addressed. We have presented some samples simulations to make the problem clear. Early experiments on James, controlling 2 DOF, have confirmed the effectiveness of the proposed approach. Simulations for the control of the 4 DOF arm are currently ongoing. In the future, the control scheme will take into account singularities and redundancies of the kinematic chain, which have been neglected for the moment, and consider also delays.

5 Acknowledgment

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