SPICE Model of Lossy Piezoelectric Polymers

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Abstract — Transmission line equivalent model for lossy piezoelectric polymers and its SPICE implementation are presented. The model includes the mechanical/viscoelastic, dielectric/electrical and piezoelectric/electromechanical losses, in a novel way by using complex elastic, dielectric and piezoelectric constants – obtained from measured impedance of PVDF-TrFE sample. The equivalent circuit parameters are derived from analogies between lossy electrical transmission line and acoustic wave propagation. The simulated impedance and phase plots of polymer, working in thickness mode, have been compared with measured data.

I. INTRODUCTION

It is advantageous to develop and implement the theoretical model of transducer in such a way that overall sensor (transducer + signal conditioning electronics) performance can be optimized. In this context, the ease with which the signal conditioning electronics can be evaluated with a SPICE like software tool makes it important to implement the theoretical model of transducer also with a similar software tool.

A SPICE implementation of piezoelectric transducer was reported by Morris et al. in [1]. The unphysical usage of negative capacitance by Morris et al. was avoided by Leach with the controlled source technique in an alternative SPICE implementation [2]. These works assumed the transducer to be lossless and hence they are insufficient for evaluating the performance of lossy transducers. Püttmer et al. [3] presented an improved model by taking into account the approximate acoustic losses with a resistor –valued fundamental resonance- in the transmission line. Püttmer et al. assumed negligible dielectric and electromechanical losses and assumed the transmission line to be low loss line - assumptions that work well for piezoceramics. A great deal of the work published on piezoelectric ceramics based transducers, does not apply when using the piezoelectric polymers because of their unique electrical and mechanical properties [4].

Piezoelectric polymers have lossy and dispersive dielectric properties and also exhibit higher viscoelastic losses. Modeling of lossy polymers like PVDF and its copolymer needs the inclusion of all the losses mentioned above. The modified equivalent model of piezoelectric polymers and its SPICE implementation presented here, is based on Leach's Model [2].

II. THEORY

Assuming a one-dimensional compression wave propagating in x direction, the thickness-mode piezoelectric transducer is shown in Fig 1. It is assumed that the electric field E and the electric displacement D are in the x direction. Let u ($=u_1-u_2$) be the net particle velocity and F ($=F_1-F_2$) be the force. If the current flowing through the external circuit is i, then charge q on the electrodes is i/s and the electric flux density D is equal to $i/(s\times A_x)$. Here, A_x is the cross-sectional area perpendicular to x axis and s ($= j\omega$) is the Laplace operator. The particle displacement is related to particle velocity by $\xi = u/s$. Thus, in the direction of wave propagation, we have:

$$\frac{dD}{dx} = 0 \Rightarrow \frac{d(i/s)}{dx} = 0 \Rightarrow \frac{d\left(h^* i/s\right)}{dx} = 0 \quad (1)$$

Using (1), the mathematical relations for the piezoelectric polymers can be written as:

$$\frac{d}{dx}\left[F - \frac{h^*}{s} i\right] = \rho A_x s u \quad (2)$$

TABLE I
DIMENSION AND COMPLEX CONSTANTS OF THE PVDF-TRFE SAMPLE

Quantitiy	Symbol	Value
Density (kg/m^3)	ρ	1880
Thickness (m)	l_x	50×10^{-6}
Width (m)	l_y	7×10^{-3}
Length (m)	l_z	7×10^{-3}
Type of Electrode	Al	Aluminum
Thickness of Electrode (A^0)	t_m	800
Electromech. Coupling const.	k_t^*	0.202-j0.0349
Piezoelectric Constant (V/m)	h_{33}^*	$3.03\times10^9 - j7.25\times10^8$
Dielectric Constant	ε^{S*}_{33}	$4.63\times10^{-11}+j8.45\times10^{-12}$
Elastic Constant (N/m^2)	c^{D*}_{33}	$1.088\times10^{10}+j5.756\times10^8$

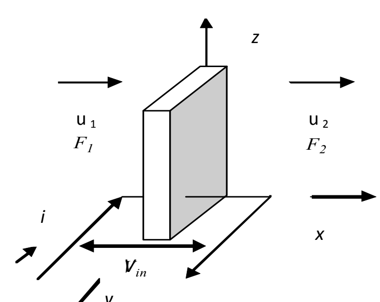

Fig. 1: Piezoelectric polymer in thickness mode.

$$\frac{du}{dx} = -\frac{s}{A_x c^*}\left[F - \frac{h^*}{s} i\right] \quad (3)$$

$$V_{in} = \frac{h^*}{s}[u_1 - u_2] + \frac{1}{C_0^* s} i \quad (4)$$

Where, V_{in} is the voltage at the electrical terminals, as shown in Fig. 1 and C_0^* is its lossy capacitance. For simplicity, the subscripts have been removed from these expressions. Eq. (2)-(3) are similar to the standard telegraphist's equations of a lossy electrical transmission line. viz:

$$\frac{dV_t}{dx} = -(L_t s + R_t) I_t \quad (5)$$

$$\frac{dI_t}{dx} = -(C_t s + G_t) V_t \quad (6)$$

Where, L_t, R_t, C_t and G_t are the per unit length inductance, resistance, capacitance and conductance of the transmission line. V_t and I_t are the voltage across and current passing through the transmission line. Comparing (2)-(3) with (5)-(6), it can be noticed that V_t is analogous to F - $(h^*/s)\times i$; L_t is analogous to $\rho \times A_x$; R_t is zero; I_t is analogous to u; and $s/(A_x \times c^*) = C_t + G_t$. The values of G_t and C_t can be obtained by substituting $s = j\omega$ and then comparing coefficients on both sides of the last expression. Thus, the lossy acoustic transmission is represented by an analogous lossy electrical transmission line. Similarly the electromechanical loss and dielectric loss are considered by using complex values of h and C_0 in (4).

Use of complex elastic, piezoelectric and dielectric constants, represented by c^*, $h^*$and ε^* respectively, ensure the inclusion of acoustic, piezoelectric and dielectric losses in the model. Piezoelectric polymers like PVDF and PVDF-TrFE have a figure of merit of 2-2.5; hence relations in IEEE standard on piezoelectricity cannot be used to find the complex elastic, piezoelectric and dielectric constants. [4]. Numerical values of these constants can be obtained by non linear regression technique [5]. The complex constants –given in Table I, were obtained from the impedance data by using "Piezoelectric Resonance Analysis Program (PRAP) [6]".

III. SPICE IMLEMENTATION

The piezoelectric polymer model has been implemented with PSPICE circuit simulator, which is commercially available from ORCAD. The SPICE schematic of the equivalent circuit is shown in Fig. 2, with clearly marked mechanical, electromechanical and electrical loss blocks.

The lossy mechanical/viscoelastic behavior of polymers is implemented with the lossy transmission line. The lossy transmission line in PSPICE also allows the use of frequency dependent expression for G_t. The frequency term in G_t is implemented by the expression *SQRT* (-*s*×*s*), where, *s* (= $j\omega$) is the Laplace operator. The parameters of transmission line viz: G_t, C_t, L_t, are given in Table II. The lossy transmission line is terminated into the acoustic impedance of the mediums on two sides of the polymer, which is air – in present work.

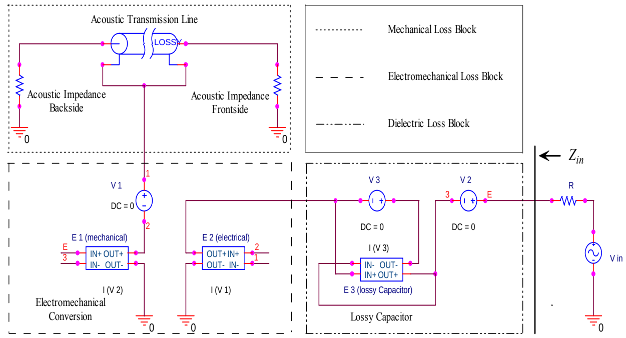

Fig. 2: PSPICE Schematic of the piezoelectric polymer model.

TABLE II
PARAMETERS OF SPICE EQUIVALENT MODEL

Quantity	Value/Expression
L_t	0.0921
G_t	$9.8958\times10^{-8}\times\sqrt{(-1\times s\times s)}$
C_t	1.8705×10^{-6}
Length	50×10^{-6} m
C_0	4.545×10^{-11}
Gain of sources E1&E2	$(3.031 \times 10^{9} - j7.252 \times 10^{8})/s$
Gain of sources E3	$1/(s \times (4.54\times10^{-11} - j\, 8.28\times10^{-12}))$

The electromechanical conversion is analogous to the transformer action and it has been implemented with behavioral modeling of controlled sources, i.e. with the ELAPLACE function of PSPICE. The currents passing through the controlled sources *E1*(*mechanical*) and *E2*(*electrical*) are (h^*/s) times the currents passing through *V2* and *V1* respectively. In SPICE, the current in any branch can be measured with a voltage source with DC value equal to zero, in that branch. As shown in Fig. 2, the voltage sources *V1* and *V2* have zero DC values and are used only to measure the current passing through them. The complex number operator *'j'* in the expression of h^*, is implemented by using the expression - *s/abs*(*s*).

The dielectric loss block consists of the lossy capacitance of polymer which is connected with external voltage source or load. Lossy capacitor is equivalent to the lossless capacitor, C_0 $(= \varepsilon A_x / l_x)$ connected in parallel with the frequency dependent resistor $R_c = (1/\omega C_0 \tan\delta_e)$. As SPICE circuit simulators allow only constant values of resistors and capacitors, the lossy capacitor with the frequency dependent resistor have been implemented with the behavior modeling of controlled voltage source, as shown in Fig 2. The negligibly small external resistance *R* shown in Fig. 2 is a necessary requirement of SPICE implementation.

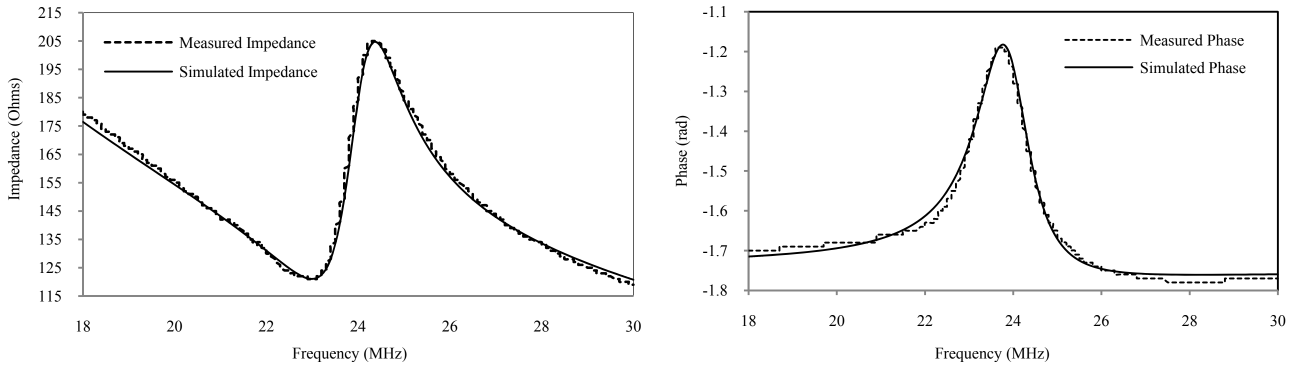

Fig. 3: Comparison of simulated Impedance (left) and Phase (right) of PVDF-TrFE with corresponding plots obtained from the measured data.

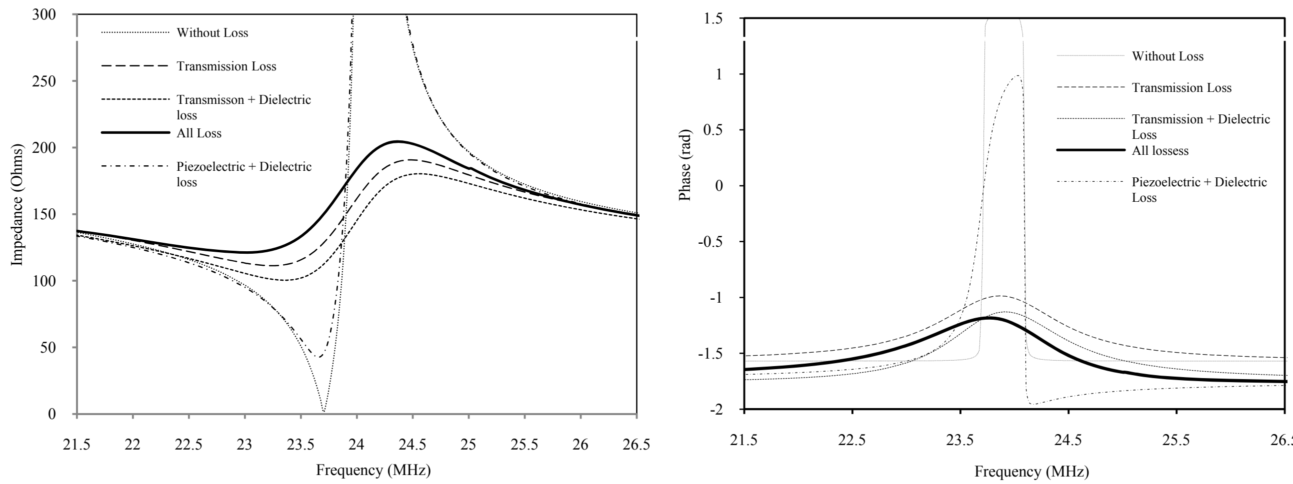

Fig. 4: Simulated impedance (right) and phase (leftt) of PVDF-TrFE sample with various combinations of losses.

IV Experiment Versus Simulation

The impedance and phase values, of a PVDF-TrFE sample were obtained with HP4285 LCR meter. The parameters of transmission line, controlled sources and the lossy capacitors were obtained by using the complex constants given in Table I. These parameters are given in Table II. Using these parameters in the SPICE model, the simulated impedance and phase of the polymer were obtained by dividing the voltage at node *E* with the current through this node. Due to their negligible thickness, the effect of electrodes is assumed to be negligible.

The simulated impedance and phase plots have been compared in Fig. 3 with corresponding plots obtained from the measured data. It can be noticed that both simulated impedance and phase plots are in good agreement with the corresponding measured values, over a wide range of frequencies around resonance. Simulated impedance and phase, assuming presence of different combination of losses, has been shown in Fig 4. It can be noticed that inclusion of only acoustic/transmission losses gives a first approximation of the impedance and phase, which is further improved by the dielectric and piezoelectric losses.

V Conclusion

A SPICE model for the thickness mode piezoelectric polymers is presented. The performance of a PVDF-TrFE sample has been evaluated against the measured data. It is observed that the presented model for lossy polymers provides a good match between simulated and measure data. With the implementation of the transducer model in SPICE it will be easier to evaluate the performance of transducer, both, in time and frequency domains.

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