Modeling of Lossy Piezoelectric Polymers in SPICE

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ABSTRACT

Piezoelectric polymers have lossy and dispersive dielectric properties and exhibit higher viscoelastic losses. Due to their lossy behavior, the lossy models developed for piezoceramics are insufficient for evaluating polymers. In this work we present a novel SPICE implementation of piezoelectric polymers model which includes the mechanical, electromechanical and dielectric losses. The mechanical/viscoelastic, dielectric/electrical and piezoelectric/electromechanical losses have been included in the model by using complex elastic, dielectric and piezoelectric constants – obtained from measured impedance of PVDF-TrFE sample. The simulated impedance and phase plots of polymer, working in thickness mode, have been compared with measured data. The impedance and phase plots have also been compared with those obtained by using the lossy model approaches reported earlier.

Keywords: Piezoelectric polymers, SPICE, Losses, Modeling

INTRODUCTION

"Smart materials" like piezoelectric polymers (e.g PVDF or its copolymer - PVDF-TrFE), are of interest in rapidly expanding range of applications (e.g. ultrasonic, MEMS etc.) as they are tough, flexible and are readily available in the form of thin films. Their close acoustic impedance match to water and living tissues is an advantage in biomedical applications. Effective implementation of a piezoelectric polymer based system directly depends on the degree to which their behavior and properties are understood. Piezoelectric polymers have lossy and dispersive dielectric properties and also exhibit higher viscoelastic losses. Much work has been published on transducers using piezoelectric ceramics, but a great deal of this work does not apply when using the piezoelectric polymers because of their unique electrical and mechanical properties [1].

It is valuable to use some form of theoretical model to assess the performance of a transducer, the effects of design changes, constructional flaws, electronics modifications etc. Instead of evaluating the transducer and conditioning electronics independently (which may not result in optimized sensor performance), it is particularly advantageous to develop and implement the theoretical model of transducer in such a way that overall sensor (transducer + conditioning electronics) performance can be optimized. In this context, the ease with which the conditioning electronics can be designed with a SPICE like software tool makes it important to implement the theoretical model of transducer also with a similar software tool. Moreover, with SPICE it is easier to evaluate the performance of transducer, both, in time and frequency domains.

Starting from the Redwood's transmission line version of Mason's equivalent circuit [2], a SPICE implementation of piezoelectric transducer was reported by Morris et al. in [3]. The unphysical usage of negative capacitance, $-C_0$ by Morris et al. was avoided by Leach with the controlled source technique in the alternative SPICE implementation [4]. The models presented in these works were verified for the transducer working in the actuating mode i.e. electrical input and mechanical output. The transducer was assumed to be lossless in these works and hence, these models are insufficient to evaluate the performance of transducers with significant losses. Püttmer et al. [5] improved these piezoceramics models by using a resistor - whose value was obtained at fundamental resonance - to accommodate the acoustic losses in the transmission line. They assumed transmission line to be having low loss and negligible dielectric and electromechanical losses in the transducer - assumptions that work well for piezoceramics. Modeling of lossy polymers like PVDF, it is requires the inclusion of all these losses. Some attempts to improve the models at system level have also been reported [6, 7] where the dispersive losses of the medium (e.g. water in the pulse-echo method) have been included in the model, but, the transducer related losses were still the same as those reported in [5].

Due to the high lossy behavior of piezoelectric polymers, even the lossy versions developed for piezoceramics are generally insufficient for evaluating their behavior. In this work, we present the equivalent model of piezoelectric polymers and its SPICE implementation, which is an extension of Leach's Model [4]. The model includes frequency dependent mechanical, electromechanical and dielectric losses present in the polymers. The model can be used both in actuating and sensing modes and for multilayered structures [8]. A comparison with the results obtained by Püttmer's approach has also been made.

THEORY

Piezoelectric Linear Constitutive relations:

According to the linear theory of piezoelectricity [9], the tensor relation between mechanical stress, mechanical strain, electric field and electric displacement is:

$$S_p = s_{pq}^E T_q + d_{kp} E_k \tag{1}$$

$$D_i = d_{iq} T_q + \varepsilon_{ik}^T E_k \tag{2}$$

Where, S_p is the mechanical strain in p direction, D_i is electric displacement in i direction, T_q is mechanical stress in q direction, E_k is the electric field in k direction, s^E_{pq} is elastic compliance at constant electric field, ε^T_{ik} is dielectric constant under constant stress, and d_{iq} is piezoelectric constant. In the event of polymer being used in the thickness mode as shown in Fig 1, the tensor relations (1)-(2) can be written as:

$$S_3 = s_{33}^E T_3 + d_{33} E_3 \tag{3}$$

$$D_3 = d_{33} T_3 + \varepsilon_{33}^T E_3 \tag{4}$$

The constants d_{33}, ε^T_{33}, and s^E_{33} are frequently found in the manufacturer's data for polarized polymers. The analysis is simpler if the variables S_3, T_3, D_3 and E_3 are arranged in the alternate way of writing the piezoelectric relation using D_3 and S_3 as independent variables. Thus the equations representing plane compression wave propagation in the x direction (3-direction or the direction of polarization) in a piezoelectric medium are:

$$T_3 = c_{33}^D S_3 - h_{33} D_3 \tag{5}$$

$$E_3 = -h_{33} S_3 + \beta_{33}^S D_3 = -h_{33} S_3 + \frac{D_3}{\varepsilon_{33}^S} \tag{6}$$

The new constants c^D_{33}, h_{33} and ε^S_{33} are related to the previous constants by:

$$\varepsilon_{33}^S = \varepsilon_{33}^T - d_{33}^2 / s_{33}^E \tag{7}$$

$$h_{33} = d_{33} / s_{33}^E \varepsilon_{33}^T \tag{8}$$

$$c_{33}^D = h_{33}^2 \varepsilon_{33}^T + 1 / s_{33}^E \tag{9}$$

Based on the choice of independent variables, there can be two other variants of (3) and (4) which are not given here. Inside polymer, the mechanical strain, mechanical stress and the electrical displacement can be written as:

$$S_3 = \frac{\partial \xi}{\partial x} \tag{10}$$

$$\frac{\partial T_3}{\partial x} = \rho \frac{\partial^2 \xi}{\partial t^2} \qquad (11)$$

$$DivD = 0 \qquad (12)$$

Where ξ is the displacement of the particles inside polymer, ρ is the density of the polymer. Eq (11) is basically Newton's law and (12) is Gauss's law. Using (5) and (10)-(12), the mechanical behavior of the particles inside polymer can be described as a wave motion which is given by:

$$\frac{\partial^2 \xi}{\partial t^2} = v^2 \frac{\partial^2 \xi}{\partial x^2} \qquad (13)$$

Where, $v = \sqrt{c_{33}^D / \rho}$ is the sound velocity in the polymer and should not be confused with particle velocity ($\partial \xi / \partial t$).

Losses:

In general, the piezoelectric polymers possess frequency dependent mechanical, dielectric and electromechanical losses. These losses can be taken into account by replacing elastic, dielectric and piezoelectric constants in (1)-(13), with their complex values [1, 10]. Thus, the mechanical/viscoelastic, dielectric and electromechanical losses are taken into account by the complex elastic constant, $c^D_{33}{}^*$; complex dielectric constant, $\varepsilon^S_{33}{}^*$ and complex electromechanical coupling coefficient, k_t^* respectively. These complex constants can be written as:

$$c_{33}^{D*} = c_r + jc_i = c_{33}^D\left(1 + j\tan\delta_m\right) \qquad (14)$$

$$\varepsilon_{33}^{S*} = \varepsilon_r - j\varepsilon_i = \varepsilon_{33}^S\left(1 - j\tan\delta_e\right) \qquad (15)$$

$$k_t^* = k_{tr} + jk_{ti} = k_t\left(1 + j\tan\delta_k\right) \qquad (16)$$

Where, the subscripts r and i stand for real and imaginary terms and tanδ_m, tanδ_e, tanδ_k are the elastic, dielectric and electromechanical coupling factor loss tangent, respectively. The complex piezoelectric constants viz: $d_{33}{}^*$ and $h_{33}{}^*$, can be obtained from electromechanical coupling constant k_t^* [9].

Polymer model with losses:

Assuming that a one-dimensional compression wave is propagating in x direction of thickness-mode piezoelectric transducer, as shown in Fig 1. In addition it is also assumed that the electric field E and the electric displacement D are in the x direction. Let u (=u_a-u_b) be the net particle velocity, F (=$F_a - F_b$) be the force, and l_x, l_y, l_z are the dimensions of the polymer. Using (5)-(6) and (10)-(12), the mathematical relations for the piezoelectric polymer can be written as:

$$\frac{dF}{dx} = -\rho A_x s u \qquad (17)$$

$$c^* \frac{d\xi}{dx} = -\frac{1}{A_x} F + h^* D \qquad (18)$$

$$E = -h^* \frac{d\xi}{dx} + \frac{1}{\varepsilon^*} D \qquad (19)$$

For simplicity, the subscripts have been removed from these expressions. In these equations, s (= $j\omega$) is the Laplace operator and A_x (= $l_z \times l_y$) is the cross-sectional area perpendicular to x axis. Complex elastic constant, piezoelectric constant and dielectric constant are represented by c^*, $h^*$and ε^* respectively. Numerical values of these constants are obtained from the impedance measurements by using non linear regression techniques [11].

If the current flowing through the external circuit is i, then charge q on the electrodes is i/s and the electric flux density D is equal to $i/(s \times A_x)$. The particle displacement is related to particle velocity by $\xi = u/s$. From (10),

$$\frac{dD}{dx}=0\Rightarrow\frac{d(i/s)}{dx}=0\Rightarrow\frac{d(h^{*}i/s)}{dx}=0 \tag{20}$$

Subtracting this expression from both sides of (17) and using above mathematical relations (17)-(18) can be written as:

$$\frac{d}{dx}\left[F-\frac{h^{*}}{s}i\right]=\rho A_{x}su \tag{21}$$

$$\frac{du}{dx}=-\frac{s}{A_{x}c^{*}}\left[F-\frac{h^{*}}{s}i\right] \tag{22}$$

$$V_{in}=\frac{h^{*}}{s}\left[u_{1}-u_{2}\right]+\frac{1}{C_{0}^{*}s}i \tag{23}$$

Where, V_{in} is the voltage at the electrical terminals of polymer and C_0^* is its lossy capacitance. Eq (21) - (22) describe the mechanical behavior and (23) describe the electromechanical conversion. It can be noted that (21)-(22) are similar to the standard telegraphist's equations of a lossy electrical transmission line viz:

$$\frac{dV_{t}}{dx}=-(L_{t}s+R_{t})I_{t} \tag{24}$$

$$\frac{dI_{t}}{dx}=-(C_{t}s+G_{t})V_{t} \tag{25}$$

Where, L_t, R_t, C_t and G_t are the per unit length inductance, resistance, capacitance and conductance of the transmission

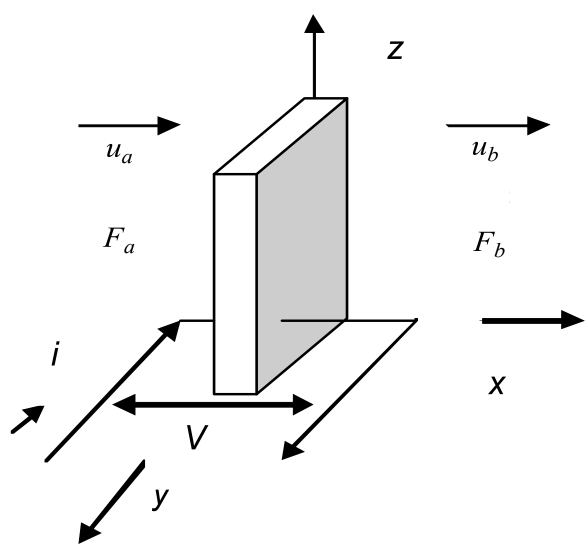

Fig 1. Piezoelectric polymer in thickness mode.

line. V_t and I_t are the voltage across and current passing through the transmission line. Comparing (21)-(22) with (24)-(25), the analogy between these two set of equations can be observed. Thus, V_t is analogous to $F\text{-}(h^*/s)\times i$; L_t is analogous to $\rho\times A_x$; R_t is zero; I_t is analogous to u; and $s/(A_x\times c^*) = C_t + G_t$. The right hand side of the last expression is a complex quantity due to complex c^*. By substituting $s = j\omega$ and then comparing coefficients on both sides, the expressions of G_t and C_t can be written as:

$$G_t = \frac{c_i \omega}{\left(c_r^2 + c_i^2\right) A_x} \tag{26}$$

$$C_t = \frac{c_r}{\left(c_r^2 + c_i^2\right) A_x} \tag{27}$$

Thus, the acoustic transmission can be represented by an analogous lossy electrical transmission line. Similarly the electromechanical loss and dielectric loss are considered by using complex values of h and C_0 in (23).

SPICE IMPLEMENTATION

Following the discussion above, the piezoelectric polymer model has been implemented with PSPICE circuit simulator, which is commercially available from ORCAD. Fig 2 shows the SPICE schematic of the piezoelectric polymer equivalent circuit. The mechanical, electromechanical and electrical loss blocks are clearly marked in Fig. 2. The SPICE implementation of various blocks is explained below.

Mechanical Loss Block:

The analogy between (21)-(22) and (24)-(25) allows the implementation of mechanical/viscoelastic behavior during acoustic transmission in polymer with a lossy transmission line, which is available in the PSPICE. It can also be implemented with lumped ladder arrangement of G_t, C_t, L_t, R_t. But, here the lossy transmission line is preferred due to the advantages - in terms of accuracy and computation time - offered by the distributed values of G_t, C_t, L_t, R_t. Further, the lossy transmission line in PSPICE allows the use of frequency dependent expression for G_t, which is desired according to (26). The frequency term in G_t is implemented in PSPICE, by using the expression $SQRT\ (-s\times s)$, where, $s\ (= j\omega)$ is the Laplace operator. The parameters of transmission line viz: G_t, C_t and L_t are obtained by using complex elastic constant i.e. c^*, from Table 1, in the analogous expression obtained from the analogy between acoustic wave propagation and the

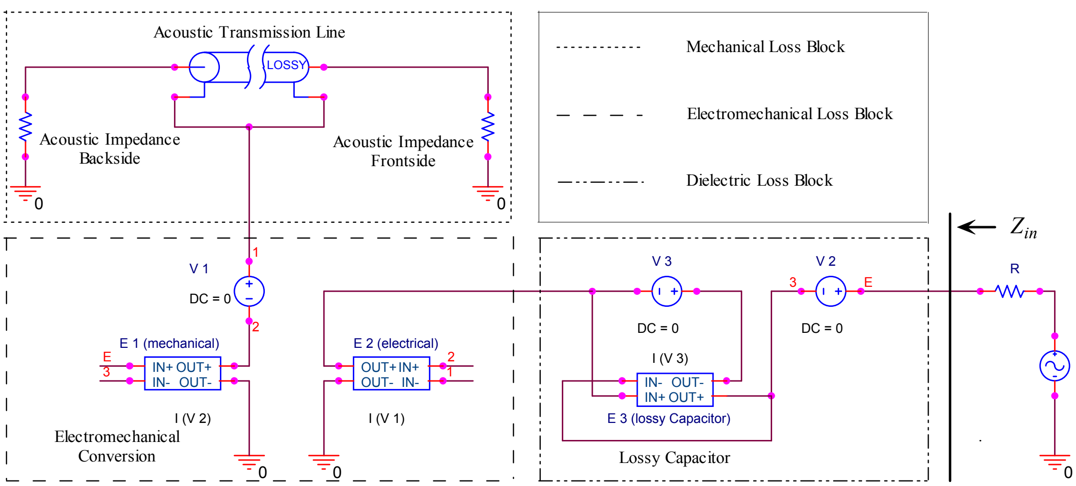

Fig. 2. Schematic of the equivalent model of piezoelectric polymer implemented with PSPICE. The model has been divided into various blocks showing the mechanical/acoustic/viscoelastic, electromechanical/piezoelectric and electric/dielectric losses.

TABLE 1

Dimension and Complex Constants of the PVDF-TrFE Sample

Quantity with Units	Symbol	Value
Density (kg/m^3)	ρ	1880
Thickness (m)	l_x	50×10^{-6}
Width (m)	l_y	7×10^{-3}
Length (m)	l_z	7×10^{-3}
Type of Electrode	Al	Aluminum
Thickness of Electrode (A^0)	t_m	800
Electromech. Coupling const.	k_t^*	0.202-j0.0349
Piezoelectric Constant (V/m)	h_{33}^*	$3.03\text{x}10^9$- $j7.25\text{x}10^8$
Dielectric Constant	$\varepsilon^{S*}{}_{33}$	$4.63\text{x}10^{-11}+j8.45\text{x}10^{-12}$
Elastic Constant (N/m^2)	$c^{D*}{}_{33}$	$1.088\text{x}10^{10}+j5.756\text{x}10^8$

lossy electrical transmission line. As shown in Fig. 2, the lossy transmission line is terminated into the acoustic impedance of the mediums on two sides of the polymer - which is air in present case. In a multilayer transducer, the acoustic impedances on both sides can be replaced by transmission lines having parameters (acoustic impedance and time delay etc.) corresponding to the mediums on each side.

Electromechanical Loss Block:

The electromechanical conversion is analogous to the transformer action. It has been implemented with the behavioral modeling of controlled sources, i.e. with the 'ELAPLACE' function in PSPICE. The currents passing through the controlled sources *E1(mechanical)* and *E2(electrical)*; are h^*/s times the currents passing through *V2* and *V1* respectively. The piezoelectric constant used in the gain term i.e. 'h^*/s' of controlled sources *E1(mechanical)* and *E2(electrical),* is given in Table 1. The complex piezoelectric constant, h^* ensures the inclusion of piezoelectric losses in the model. The complex number operator 'j' in the expression of h^* is implemented in PSPICE by using the expression -s/abs(s).

Dielectric Loss Block:

This block consists of the lossy capacitance of polymer connected to the external voltage source or load. The use of complex permittivity i.e. ε^*, ensures the inclusion of dielectric losses. The lossy capacitor obtained by using ε^*, is equivalent to a lossless capacitor, $C_0 = \varepsilon\times A_x/l_x$ connected in parallel with the frequency dependent resistor R_0=1/ (ωC_0 tanδ_e), where, C_0 is the static lossless capacitance of polymer. The voltage across the equivalent lossy capacitor is given as:

$$V_c = \frac{I_c}{sC_0 + \omega C_0 \tan\delta_e} \tag{28}$$

The SPICE circuit simulators allow only constant values of resistors and capacitors. To implement the lossy capacitor - which has the frequency dependent resistor - the behavior modeling of controlled voltage source has been used in this work. The controlled source *E3(*lossy *capacitor)* is implemented with the 'ELAPLACE' function of PSPICE. The dielectric constant used in the expression of the gain term of controlled source *E3(lossy capacitor),* is given in Table 1. Again, ω is implemented with the expression *SQRT* ($-s\times s$).

EXPERIMENT VERSUS SIMULATION

The impedance and phase values, of a PVDF-TrFE sample were obtained with HP4285 LCR meter. Following (23)-(28), the parameters of transmission line, controlled sources and the lossy capacitors were obtained by using the complex constants given in Table 1. Since piezoelectric polymer have figure of merit < 5, the method given in [9] cannot be used to obtain the material constants [1]. The complex constants given in Table 1 were thus obtained with the PRAP (Piezoelectric Resonance Analysis Program) [12] analysis of the measured impedance data. Using these parameters in

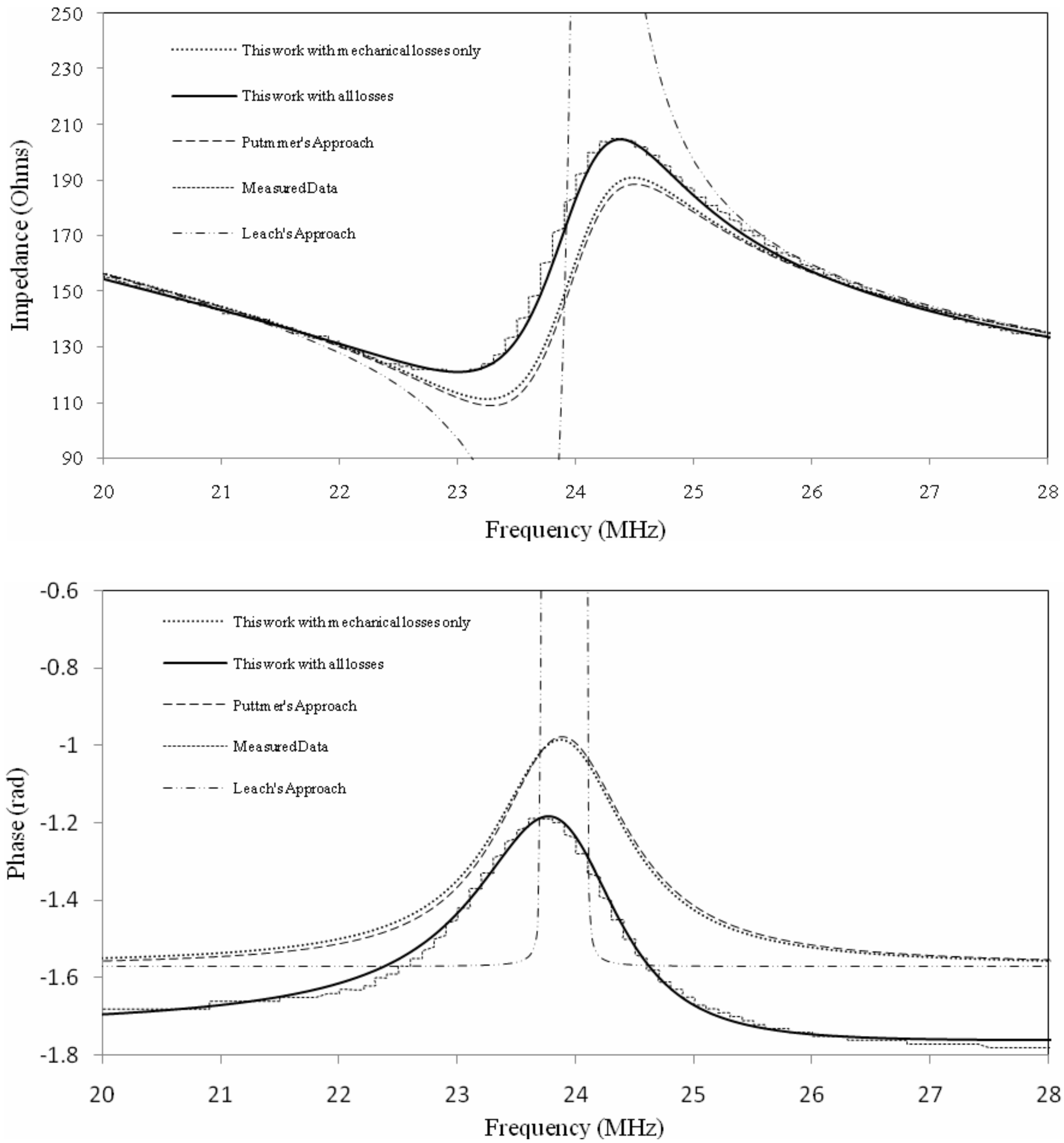

Fig. 3. Comparison of Impedance (top) and Phase (bottom) of the PVDF-TrFE sample.

the SPICE model, the simulated impedance and phase of the polymer were obtained by dividing the voltage at node *E* with the current through this node. Due to their negligible thickness, the effect of electrodes is assumed to be negligible.

The simulated impedance and phase plots have been compared in Fig. 3 with corresponding plots obtained from the measured data. It can be noticed that both simulated impedance and phase plots are in good agreement with the corresponding plots obtained from measured values, over a wide range of frequencies around resonance. The impedance and phase plots have also been compared with corresponding plots obtained from Leach's approach and Püttmer's approach. Leach's approach assumes all losses to be negligible and Püttmer's approach considers only transmission losses. In case of Püttmer's approach, the losses in the transmission line are represented by *'R_t'*, whereas in the work presented here; these losses are represented by *'G_t'*. A comparison of plots obtained from Püttmer's approach with those obtained by the approach presented in this work and having only transmission line losses, is also shown in Fig 3. The close agreement among these plots shows that acoustic transmission loss can be represented by either 'R_t' or 'G_t'. The improved match of plots obtained with the approach presented here, with those obtained from the measured data, can be clearly noticed in Fig 3.

CONCLUSION

A SPICE model for the thickness mode piezoelectric polymers is presented. The performance of a PVDF-TrFE sample has been evaluated against the measured data and against earlier reported approaches. It is observed that the model for lossy polymers presented here provides a better match with the measured data than those reported earlier. With the implementation of the transducer model in SPICE it will be easier to evaluate the performance of transducer, both, in time and frequency domains.

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